**Question 1**

1. (Liu 2020a)
2. (Liu 2020a)
3. (Liu 2020a)
4. (Liu 2020a)

**Question 2**

1. (Liu 2020a)
2. (Liu 2020a)
3. (Liu 2020a)
4. (Liu 2020a)

**Question 3**

(Liu 2020a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 |

(Liu 2020a)

**Question 4**

1. **Antisymmetric**, as if person is older than person , then person is not older than person .  
   **Transitive**, because if person is older than person and person is older than person , then person is older than person .  
   (Rosen 2007)
2. **Reflexive**, as if person was born on the same day as person , then the day person was born on is equal to itself.

**Symmetric**, because if person was born on the same day as person , then person was born on the same day as person .  
**Transitive**, as if person was born on the same day as person and person was born on the same day as person , then person was born on the same day as person .

(Rosen 2007)

1. **Reflexive**, as if person has the same first name as person , then person ’s first name is equal to itself.

**Symmetric**, because if a person has the same first name as person , then person has the same first name as person .  
**Transitive**, as if person has the same first name as person and person has the same first name as person , then person has the same first name as person .

(Rosen 2007)

1. **Reflexive**, as if person has a grandparent in common with person , then person ’s grandparent is in common with itself.

**Symmetric**, because if a person has a grandparent in common with person , then person has a grandparent in common with person .

(Rosen 2007)

**Question 5**

1. contains all ordered pairs , such that student is either required to read book in a unit or has read book .

(Rosen 2007)

1. contains all ordered pairs , where is a student who is required to read book in a unit but has not read it or needs to read book but has not read it.   
   (Rosen 2007)
2. is the set of all ordered pairs , such that student is required to read book in a unit but has not read it.

(Rosen 2007)

1. contains all ordered pairs , such that student has read book but is not required to read in a unit.

(Rosen 2007)

**Question 6**

1. Since contains elements (and, in turn, its length is ), the matrix representing , will be of order and will be defined by if is in and otherwise. Moreover, in order to find , we need to interchange the rows and columns of . Therefore, since the entries of only have their positions changed, the number of nonzero entries in will be given by the number of nonzero entries in (i.e. ).  
   (Rosen 2007)
2. Since contains elements (and, in turn, its length is ), the matrix representing , will be of order and will be defined by if is in and otherwise. Moreover, in order to find , we need to change each 0 and 1 in to 1 and 0, respectively. Therefore, the number of nonzero entries in will be given by the number of entries in less its number of nonzero entries   
   (i.e. ).   
   (Rosen 2007)

**Question 7**

Definition: A relation on a set is called an equivalence relation if it is reflexive, symmetric and transitive

1. **Reflexivity**i.e. Show that

Since , is reflexive

**Symmetry**

i.e. Show that

Since , is symmetric

**Transitivity**

i.e. Show that

Since , is transitive

Since is reflexive, symmetric and transitive, it is an equivalence relation

(The Math Sorcerer, 2018)

1. (Rosen 2007)

**Question 8**

Definition: A relation on a set is called partial order if it is reflexive, antisymmetric and transitive (Rosen 2007).

1. **Reflexive**, as each vertex in the digraph features an arc in the form (i.e. a loop)  
   **Antisymmetric**, as each vertex in the digraph features an arc in the form , but not an arc in the form   
   Not **Transitive**, as there is an edge from a to b and an edge from b to c, but there is no edge from a to c.   
   (Liu 2020b)
2. **Reflexive**, as each vertex in the digraph features an arc in the form (i.e. a loop)  
   **Antisymmetric**, as each vertex in the digraph features an arc in the form , but not an arc in the form   
   **Transitive**, as whenever there is an edge and an edge , there is an edge   
   (Liu 2020b)

**Question 9**

1. For any bits, there are possible bit strings. If a string is length , then we can write its length as being either , if is even or , if is odd, where

Regardless, the bits of the first half of the bit string determine the bits of the second half. Therefore, let

be a bit string that is a palindrome.

Therefore, there are 2 choices ( or ) for each where . Furthermore, if is odd, then there are also 2 choices for . Resultantly, we see that if is even, we have a total of bit strings and if is odd, we have a total of . Therefore, if is even then we have a total of palindromic sub strings and if is odd then we have a total of .

(Teja713 2015)

Let’s assume that “contains two consecutive 0s” implies that a bit string contains two *and only two* consecutive 0s (i.e. one set of two consecutive 0s), as the question does not specify if we should take into account only bit strings with one and only one set of two consecutive 0s or all bit strings with at least one set of consecutive 0s. Since the length of the bit string is relatively small, we can determine the solution manually. Therefore, let’s place 2 consecutive 0s at all indexes, marking bits at all other indexes with x (to represent either 0 or 1) and then determine the number of bit strings with a set of consecutive zeros. Let be the number of bit strings that contain one and only one set of two consecutive 0s:  
  
 The number of bit strings that are palindromes if the bit string is of length 7 and contains two consecutive 0s is .

1. Let’s assume that “containing three consecutive 1s” implies that a bit string contains three *and only three* consecutive 1s (i.e. one set of three consecutive 1s), as the question does not specify if we should take into account only bit strings with one and only one set of three consecutive 1s or all bit strings with at least one set of consecutive 1s. Since the length of the bit string is relatively small, we can determine the solution manually. However, it would be easier to determine the solution by subtracting the number of bit strings which contain three consecutive 0s from the total number of bit strings. Let be the number of bit strings that contain one and only one set of three consecutive 1s:

The number of bit strings that are palindromes if the bit string is of length 8 and

does not contain three consecutive 1s is

**Question 10**

1. Firstly, we need to seat people at the 8-seater table. However, we first need to select 8 people to seat at that table. The number of ways in which we can select 8 people from the original 15 is given by .  
     
   Next, we need to seat the 8 people we selected around the 8-seater table. The number of ways in which we can seat these 8 people is given by . Given that “seatings are considered to be the same if they can be obtained from each other by rotating the table” we needed to discount arrangements which can be obtained by rotating a particular arrangement. Hence, the reason for .  
     
   Thereafter, we need to seat the remaining 7 people around the 7-seater table. The number of ways we can seat these 7 people is given by . Again, the reason for is the same as above.

Consequently, the number of ways is given by   
(Liu 2020c)

1. Firstly, we need to seat people at the 8-seater table. However, we first need to select 8 people to seat at that table. Given that 2 particular people have to sit next to each other, we can treat these 2 people as 1. Therefore, we need to select 7 people and thus, the number of ways in which we can select 7 people from the original 15 is given by .  
     
   Next, we need to seat the 7 people we selected around the 8-seater table, where the “7” selected includes of 2 people that have to sit next to each other being counted as 1. The number of ways in which we can seat these 7 people is given by . Given that “seatings are considered to be the same if they can be obtained from each other by rotating the table” we needed to discount arrangements which can be obtained by rotating a particular arrangement. Hence, the reason for . Additionally, we need to consider the number of permutations of persons a and b. Hence, .

Thereafter, we need to seat the remaining 7 people around the 7-seater table. The number of ways we can seat these 7 people is given by .

Consequently, the number of ways is given by   
(Liu 2020c)

1. …

**Question 11**

1. Let be the total number of possible passwords, and and denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule . We will now find and . Using the sum and product rules, we obtain the number of passwords of characters:

Then using the general formula for calculating , we calculate :

Let be the time taken, in years, to try every password.

(matin 2014; Rosen 2007)

1. Let be the total number of possible passwords which do not contain any of the six special characters and and denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule . We will now find and . Using the sum and product rules, we obtain the number of passwords of characters:

Then using the general formula for calculating , we calculate :

Finally, Let be the total number of possible passwords which contain at least one of the six special characters:

(Rosen 2007)

1. Let be the total number of possible passwords and and denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Furthermore, let be the total number of possible passwords which contain at least one uppercase letter, one lowercase letter, one digit and one special character and and denote the number of possible passwords of length 8, 9, 10, 11 and 12, respectively. Using the sum rule . We will now find and . Using the sum and product rules, we obtain all of the number of passwords of characters:

Then using the general formula for calculating , we get

Thereafter, we remove all passwords with no lowercase letter (), all passwords with no uppercase letter (), all passwords with no digit (), and all passwords with no special character ().  
  
However, we have removed some passwords multiple times. Thus, we must add back all passwords with:

* No lowercase letter and no uppercase letter ()
* No lowercase letter and no digit ()
* No lowercase letter and no special character ()
* No uppercase letter and no digit ()
* No uppercase letter and no special character ()
* No digit and no special character ()

However, we have added back some passwords multiple times. For example, an all-digit password was removed three times in the first step, then put back three times in the second step. Thus, it must be removed again:

* Only lowercase letters ()
* Only uppercase letters ()
* Only digits ()
* Only special characters ()

Then, using the above we obtain the number of passwords of 8 characters which contain at least one uppercase letter, one lowercase letter, one digit and one special character:

Similarly,

and

and

and

Consequently,

(user2192774 2014)

**Question 12**

1. Let be the event that a person has COVID-19 and be the event that this person tests positive for the disease. Consequently, we want to determine and to do so, we use Bayes’ Theorem. In order to determine we need to find , , and .  
     
   We know that the chance of having COVID-19 is 0.0001, therefore, and . Because someone who has COVID-19 tests positive 99% of the time, we know that ; this is probability of a true positive (i.e. that someone with COVID-19 tests positive). We also know that ; this is the probability of a false negative (i.e. that someone with COVID-19 tests negative). Furthermore, because someone who does not have COVID-19 tests positive 0.2% of the time, we know that ; this is the probability of a false positive (i.e. that someone without COVID-19 tests positive).

The probability that someone who tests positive for COVID-19 actually has the disease is given by . By Bayes’ Theorem, we know that

This means that only 5% of people who test positive for COVID-19 actually have the disease. Because COVID-19 is rare, the number of false positives from the diagnostic test is greater than the number of true positives, making the percentage of people who test positive who actually have the disease quite small. People who test positive for COVID-19 should be relatively concerned that they actually have the disease.   
(Rosen 2007)

1. The probability that someone who tests negative for COVID-19 has the disease is given by . By Bayes’ Theorem, we know that

Consequently, 5% of people who test negative have COVID-19.  
(Liu 2020d; Rosen 2007)

**Question 13**

1. The information can be summarised in a tabular format, where the entry gives the probability that is received, given that was sent:

|  |  |  |
| --- | --- | --- |
|  | Received | Received |
| Sent |  |  |
| Sent |  |  |

Let be the probability that is received, be the fact that was sent and be the fact that was received.

(Department of Computer Science 2009)

1. i.e. Determine By using Bayes’ theorem, we know that

(Department of Computer Science 2009)

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